## MULTIPLE CHOICE SOLUTIONS--MECHANICS

## TEST I

1.) The position function for an oscillating body is $x=20 \sin (.6 t-\pi / 2)$. At $t=0$, the magnitude of the body's acceleration is:
a.) $20 \mathrm{~m} / \mathrm{s}^{2}$. [The acceleration function is $\mathrm{a}=-\omega^{2} \mathrm{~A} \sin (\omega \mathrm{t}+\delta)$. Putting in the numbers sans' units yields $a=-(.6)^{2}(20) \sin \left[(.6(0)-\pi / 2)\right.$, or $-7.2 \mathrm{~m} / \mathrm{s}^{2}$. The magnitude is $7.2 \mathrm{~m} / \mathrm{s}^{2}$, and this response is false.]
b.) $12 \mathrm{~m} / \mathrm{s}^{2}$. [Nope.]
c.) $7.2 \mathrm{~m} / \mathrm{s}^{2}$. [This is the one.]
d.) None of the above. [Nope.]
2.) A satellite following an elliptical path around a planet has an angular velocity $\omega_{f a r}$ when at its maximum distance $d$ units from the planet's center. At its closest point, the distance between the satellite and planet's center is $d / 3$. The satellite's angular velocity at that closest point is:
a.) $\omega_{\text {far }} / 3$. [As there are no external torques acting on the system, angular momentum is conserved. There are two ways to do this: the hard way and the short way. Both are educational, so we will try each one. Looking at the body as an object moving in a circular motion, we can write the conservation of angular momentum in angular terms as: $I_{f a r} \omega_{\text {far }}=I_{\text {near }} \omega_{\text {near }}$, or $\left(m R_{f a r}{ }^{2}\right) \omega_{\text {far }}$ $=\left(m R_{\text {near }}{ }^{2}\right) \omega_{\text {near. }} . \quad$ Setting $R_{n e a r}=d / 3$ and $R_{\text {far }}=d$, we get $\omega_{\text {near }}=9 \omega_{\text {far }}$. An alternative is to look at the body's translational motion. We can write the conservation of angular momentum as $\boldsymbol{R}_{\text {far }} x \boldsymbol{p}_{\text {far }}=\boldsymbol{R}_{\text {near }} x \boldsymbol{p}_{\text {near. }}$. At the near and far points, the angle between $\boldsymbol{R}$ and $\boldsymbol{p}$ is $90^{\circ}$, so the cross products become $(\mathrm{p})(\mathrm{R}) \sin 90^{\circ}$ terms and we can write $\left(m v_{\text {far }}\right) d=\left(m v_{n e a r}\right)(d / 3)$, or $\mathrm{v}_{\text {near }}=3 v_{\text {far }}$. Noting that $v_{f a r}=d \omega_{f a r}$ and $v_{\text {near }}=(d / 3) \omega_{n e a r}$, we can rewrite $v_{n e a r}=3 v_{f a r}$ as $(d / 3) \omega_{\text {near }}=$ $3(d) \omega_{\text {far }}$, or $\omega_{\text {near }}=9 \omega_{\text {far }}$. By the way, what is really interesting to note here is that even though the velocity goes up by a factor of 3 , the angular velocity goes up by a factor of 9 --not something you might have expected.]
b.) $\omega_{\text {far }}$. [From above, this is not true.]
c.) $3 \omega_{\text {far }}$ [Nope.]
d.) $9 \omega_{\text {far }}$ [Yup.]
3.) Assume $M$ is not fixed to the ground and can move frictionlessly. Assume also that the frictional force $f$ is removed from the system (i.e., the slide area across $D$ is now frictionless). The spring is released accelerating $m$.
a.) Momentum should be conserved in the system
 because the spring applies a conservative force. [Nope. Conservative forces are associated with energy considerations. Conservation of momentum is associated with external impulses (read this external forces) acting on the system. An

EXTERNAL FORCE can be a CONSERVATIVE FORCE (gravity is often external to a system, but it is a conservative force) as well as non-conservative (friction between two bodies within a system can be an internal force, but friction is the quintessential non-conservative force). This response is false.]
b.) Just after it leaves the spring, $m$ 's velocity will be $\left(k x^{2}\right) /\left(m+m^{2} / M\right)$. [In typical A.P. fashion, this question was designed to be tricky. Note that although the general expression for the potential energy function for a spring is $.5 \mathrm{kx}^{2}$, the spring's initial displacement in this particular problem is not $x$ but rather $d$. In other words, there should be no $x$ term in the answer. If you noticed this, you realized that this response must be false. If you didn't, you undoubtedly went through the process of solving. If you solved you most probably proceeded as follows: Both energy and momentum are conserved in this situation. The initial momentum was zero, so we can write: $0=m v_{1}-M v_{2}$, or $v_{2}=\left(m v_{1} / M\right)$. Energy is conserved, so you might be nudged to write: $.5 k x^{2}=.5 m v_{1}^{2}+.5 M v_{2}{ }^{2}$. Plugging in for $v_{2}$ yields: $.5 k x^{2}=.5 m v_{1}{ }^{2}+$ $.5 M\left[m v_{1} / M\right]^{2}$. Solving yields: $v_{1}=\left(k x^{2}\right) /\left(m+m^{2} / M\right)$. Again, the only thing wrong with this analysis is the fact that the problem does not deal with $x$ 's, it deals with $d$ 's.]
c.) When $m$ reaches the second spring, it will depress it a distance $d$ before coming to rest relative to the spring. Also, at that point $M$ will have come to rest. [Energy is conserved in this situation. When $m$ is shoved up against the first spring, there is spring potential energy in the system. All of that energy will be reclaimed as potential energy when $m$ has fully depressed the second spring. Momentum is conserved in the system and is equal to the system's initial momentum, or zero. That means that when $m$ is stationary with respect to $M$ (i.e., when $m$ has fully depressed the second spring), the velocity of both $m$ and $M$ must also be zero. This response is true.]
d.) All of the above. [Nope.]
4.) All three masses are released at once. Mass $m_{2}$ travels the same distance as does mass $m_{3}$. The mass with the largest velocity at
 the bottom will be:
a.) Mass $m_{1}$. [From energy considerations and no friction in the system, the kinetic energy at the bottom of the incline must equal the potential energy at the top. Because the three bodies have different masses, it is tempting to assume that the velocities will be different. Using the math, though, yields another story. Specifically: $m g h=.5 m v^{2}$, or $v=(2 g h)^{1 / 2}$. The velocity has nothing to do with the mass (this shouldn't be terribly surprising, considering the fact that two different masses allowed to fall from the same height should accelerate at the same rate--at $g$-- reaching the ground at the same time). In other words, if all three masses start from rest at the same height in a frictionless situation, they should all have the same velocity at the bottom. This response is false.]
b.) Mass $m_{2}$. [Nope.]
c.) Mass $m_{3}$. [Nope.]
d.) Masses $m_{2}$ and $m_{3}$. [Nope.]
e.) They will all have the same velocity at the bottom. [This statement is true.]
5.) Looking at the sketch in \#4, friction is introduced into the system such that the coefficient of friction is the same for all three inclines. The mass with the largest velocity at the bottom will be:
a.) Mass $m_{1}$. [In this case, the body with the largest velocity will be the one that has
had the least amount of energy removed from it by friction. That will be the one that travels the shortest distance AND has the smallest frictional force applied to it. As the frictional force is proportional to the normal force, that means we want the incline that, on average, applies the least normal force on the body (see note at the end of this Solutions section for more about this). That combination of requirements is met by $m_{3}$. This response is false.]
b.) Mass $m_{2}$. [Nope.]
c.) Mass $\boldsymbol{m}_{\boldsymbol{3}}$. [This statement is true.]
d.) Masses $m_{2}$ and $m_{3}$. [Nope.]
e.) They will all have the same velocity at the bottom. [Nope.]

## 6.) Point $A$ and Point $B$ on a wave are observed to be $7 / 4$

 wavelengths apart. At a particular instant, Point $A$ is below the axis and moving upward toward equilibrium. At that time, Point B is:a.) Below the axis and moving upward toward equilibrium. [The best way to determine what is actually happening in this situation is to draw a sine wave, place a point below the axis and moving upward (toward equilib-rium-- see Point $A$ in the sketch), and see what is happening

(below the axis
and moving upward) $7 / 4$ wavelengths away. The sketch does this, with the exception that two wavelengths ( $8 / 4$ wavelengths) has been identified for ease. From there it should be obvious that the $7 / 4$ wavelengths will put a point that is below the axis moving away from equilibrium. From the diagram, this response is false.]
b.) Below the axis and moving downward away from equilibrium. [From above, true.]
c.) Above the axis and moving upward away from equilibrium. [Nope.]
d.) Above the axis and moving downward toward equilibrium. [Nope.]
7.) A single, constant force is applied to a body. After the force does 20 joules of work, the body's velocity has changed from zero to $6 \mathrm{~m} / \mathrm{s}$. The work required to change the body's velocity from $6 \mathrm{~m} / \mathrm{s}$ to $12 \mathrm{~m} / \mathrm{s}$ is:
a.) 20 joules. [There is really no clever way to do this problem but longhand. Using the information you know, you can write: $W_{\text {net }}=K E_{2}-K E_{1}=.5 m\left(v_{2}{ }^{2}-v_{1}{ }^{2}\right)$, or $(20$ joules $)=.5 m\left(6^{2}\right.$ $0^{2}$ ), or $m=1.111 \mathrm{~kg}$. Using this information to analyze the second situation, we get: $W_{n e t}=$ $K E_{2}-K E_{1}=.5(1.111 \mathrm{~kg})\left(12^{2}-6^{2}\right)=60$ joules. This statement is false.]
b.) 40 joules. [From above, this statement is false.]
c.) 60 joules. [From above, this statement is true.]
d.) 400 joules. [From above, this statement is false.]
8.) A man on a unicycle pedaling northward begins to slow. The direction of the cycle's angular acceleration vector will be:
a.) North. [By definition, the direction of any angular acceleration vector will always coincide with the direction of the axis about which the rotation occurs. The axis of rotation for a bike moving northward will be east/west. But which is it, east or west? Think about the direction of the angular velocity. If you make the fingers of your right hand curl to mimic the wheel's motion (i.e., curl the fingers to follow the rotation of the wheel), your right-hand thumb will point west. That is the direction of the angular velocity of the wheel. If the angular acceleration's direction were also to the west, the sign of the angular velocity and angular acceleration would be the same and the bike would be speeding up. But the bike is slowing down, so the direction of the angular acceleration must be opposite west, or east. This response is false.]
b.) South. [Nope.]
c.) East. [Yup.]
d.) West. [Nope.]
e.) Upward, relative to the ground. [Nope.]
9.) A mass $m$ has a force $F$ applied to it as shown to the right:
a.) The normal force on the block will be $m g$. [There is a component of $F$ in the
 normal direction. N.S.L. yields $N-m g+F \sin \theta=m a_{y}=0$. This statement is false.]
b.) The acceleration of the block will be a function of $F$ only. [The component of $F$ that accelerates the block is a function of the angle $\theta$. This statement is false.]
c.) If there was kinetic friction acting on the block, it would be directed toward the left. [This is tricky. Because $F$ is to the right, one might assume that the motion was to the right. If that were the case, this statement would be true. Unfortunately, nothing was said about the direction of motion (just because $F$ is to the right doesn't mean the body is moving to the right--it could be slowing down while moving to the left). Because the statement could be false, it can-not be true.]
d.) None of the above. [Yup.]
10.) A graph of a traveling wave as seen at $t=2$ seconds and $t=$ 3 seconds is shown to the right. The period of the wave is:
a.) (3/4) seconds/cycle. [The period tells you how long it takes for a wave to pass by. The wave we are dealing with here moves 18 meters in 1 second (look at the graph and the distance between the position at $t=2$ seconds and $t=3$ seconds) for a wave velocity of $18 \mathrm{~m} / \mathrm{s}$. The wavelength is 24 meters (again, look at the graph--the distance between two peaks is 24 meters). Using the relationship $v=\lambda v$, we find that the frequency is $3 / 4$ cycles/second. The inverse of that (i.e., the period) is $4 / 3$ seconds per cycle. This response is false.]
b.) (4/3) second/cycle. [This is the one.]
c.) 4 seconds/cycle. [Nope]
d.) Can't tell with the information given. [Nope.]
e.) None of the above. [Nope.]
11.) A mass $m$ sits on a frictionless incline plane of angle $\theta$.
a.) The acceleration of $m$ will be $g$. [It isn't gravity that accelerates the body, it's the component of gravity along the line of the incline that accelerates the body. This statement is false.]
b.) The acceleration of $m$ will be dependent upon the size of the mass. [If you do the math, the mass terms cancel out. If you simply think about the interplay between inertial mass and gravitational mass, it should be obvious that the amount of mass will have no effect on the outcome. This statement is false.]
c.) The acceleration of $m$ will be dependent upon $\theta$. [The component of gravity that accelerates the mass will be a function of the incline's angle. This statement is true.]
d.) All of the above. [Obviously not true.]
12.) It is known that a single, conservative force that runs along the $x$ axis does -25 joules of work on a body moving in the negative $x$-direction.
a.) The force is in the negative direction, and the body's change of potential energy is -25 joules. [Negative work implies that energy is being pulled out of the system and the body is slowing down. If the body is moving in the negative direction and additionally slowing down, the force must be in the positive direction. From that observation, this statement is false.]
b.) The force is in the negative direction, and the body's change of potential energy is 25 joules. [This is false for the same reason stated in Response a.]
c.) The force is in the positive direction, and the body's change of potential energy is - 25 joules. [This has the correct force direction. As for the potential energy change, we know that $W_{\text {consforce }}=-\Delta \mathrm{U}$. In this case, then, -25 joules $=-\Delta \mathrm{U}$, or the potential energy change is +25 joules. Wrong sign for this response. This statement is false.]
d.) The force is in the positive direction, and the body's change of potential energy is 25 joules. [This statement is true, given what has been said in Responses $a$ and c.]
e.) None of the responses make any sense as gravity is not oriented along the $x$ axis. [No, gravity is NOT the only possible conservative force with a potential energy function associated with it. This statement is false.]
13.) A satellite following an elliptical path around a planet has a velocity $v_{f a r}$ when at its maximum distance $d$ units from the planet's center. At its closest point, the distance between the satellite and planet's center is $d / 3$. The satellite's velocity at that closest point is:
a.) $\mathrm{v}_{\text {far }} / 3$. [As there are no external torques acting in the system, we can use conservation of angular momentum here. There are two ways to do this: the hard way and the short way. Each is educational, so we will try both. We will write the conservation of angular momentum in angular terms as: $I_{f a r} \omega_{f a r}=I_{\text {near }} \omega_{\text {near }}$, or $\left(m R_{\text {far }}{ }^{2}\right) \omega_{\text {far }}=\left(m R_{\text {near }}{ }^{2}\right) \omega_{\text {near. }}$. Setting $R_{\text {near }}=d / 3$ and $R_{\text {far }}=$ $d$, we get $\omega_{\text {near }}=9 \omega_{\text {far }}$. Noting that $\omega_{f a r}=v_{f a r} / d$ and $\omega_{\text {near }}=v_{\text {near }} /(d / 3)$, we can rewrite $\omega_{\text {near }}=$ $9 \omega_{f a r}$ as $v_{\text {near }} l(d / 3)=9\left[v_{f a r} / d\right]$, or $v_{n e a r}=3 v_{f a r}$. This is the hard way. Looking at the body as a point mass, we can write the conservation of angular momentum as $\boldsymbol{R}_{f a r} x \boldsymbol{p}_{f a r}=\boldsymbol{R}_{n e a r} x \boldsymbol{p}_{n e a r}$. At the near and far points, the angle between $\boldsymbol{R}$ and $\boldsymbol{p}$ is $90^{\circ}$, so the cross products take the form (p)(R)sin $90^{\circ}$. The angular momentum expression becomes $\left(m v_{f a r}\right) d=\left(m v_{n e a r}\right)(d / 3)$, or $v_{n e a r}=3 v_{f a r}$. As a side note, what is really interesting to note here is that even though the velocity goes up by a factor of 3 , the angular velocity goes up by a factor of 9 --not something you might have expected.]
b.) $v_{\text {far }}$. [From above, this is not true.]
c.) $3 \mathbf{v}_{\text {far }}$. [This is the one.]
d.) $9 \mathrm{v}_{\text {far }}$. [Nope.]
14.) A turntable has an angular velocity of $.8 \mathrm{rad} / \mathrm{sec}$. It is accelerated at a rate of $-4 \mathrm{rad} / \mathrm{sec}^{2}$.
a.) After 5 seconds, its angular velocity is $-19.2 \mathrm{rad} / \mathrm{sec}$, and it will have rotated through a net angular displacement of - 46 radians. [For the first part of the response, rotational kinematics maintains: $\quad \alpha=\frac{\omega_{2}-\omega_{1}}{\mathrm{t}} \Rightarrow \omega_{2}=\omega_{1}+\alpha \mathrm{t}=(.8 \mathrm{rad} / \mathrm{sec})+\left(-4 \mathrm{rad} / \mathrm{sec}^{2}\right)(5 \mathrm{sec})=-19.2$ radians. The first part is OK. For the second part of the response, kinematics again maintains: $\Delta \theta=\omega_{1} t+.5 \alpha t^{2}=(.8$ $\mathrm{rad} / \mathrm{sec})(5 \mathrm{sec})+.5\left(-4 \mathrm{rad} / \mathrm{sec}^{2}\right)(5 \mathrm{sec})^{2}=46$ radians. This response is true, but are there others?]
b.) It will take half as long to go from $.8 \mathrm{rad} / \mathrm{sec}$ to $.4 \mathrm{rad} / \mathrm{sec}$ as it takes to go from .8 $\mathrm{rad} / \mathrm{sec}$ to $0 \mathrm{rad} / \mathrm{sec}$. [Think about the relationship between translational velocity and time. With constant, negative acceleration, doubling the time will double the velocity change. Keeping to our rotation/translation parallel, the same must be true of the relationship between time and angular velocity. If you don't believe, do the problem: $\alpha=\left(\omega_{2}-\omega_{1}\right) / \mathrm{t}, \Rightarrow \mathrm{t}=\left(\omega_{2}-\omega_{1}\right) / \alpha$. Going from $.8 \mathrm{rad} / \mathrm{sec}$ to $.4 \mathrm{rad} / \mathrm{sec}$ yields $t=(.4 \mathrm{rad} / \mathrm{sec}-.8 \mathrm{rad} / \mathrm{sec}) /\left(-4 \mathrm{rad} / \mathrm{sec}^{2}\right)=.1 \mathrm{~second}$. Going from $.8 \mathrm{rad} / \mathrm{sec}$ to $0 \mathrm{rad} / \mathrm{sec}$ yields $t=(0 \mathrm{rad} / \mathrm{sec}-.8 \mathrm{rad} / \mathrm{sec}) /\left(-4 \mathrm{rad} / \mathrm{sec}^{2}\right)=.2 \mathrm{~seconds}$. The time is doubled and this response is true.]
c.) It will have moved through a greater net angular displacement after the first . 2 seconds than after the first . 4 seconds. [The body is slowing down (as was the case with translational systems, if the signs of the velocity and acceleration terms differ, the body must be slowing down). According to the calculation in Response $b$, the time it takes for the body to slow to zero will be .2 seconds. During that period, the body will have traveled some net angular distance $\Delta \theta$. In .4 seconds, the body will have rotated to a stop after covering a net angular distance $\Delta \theta$ in the first .2 seconds, then will have retraced its steps back to its original angular position during the second .2 seconds. In other words, the body's net angular displacement after .4 seconds will be zero, and this statement is true.]
d.) Both $a$ and $b$. [This is false.]
e.) All of the above. [This is the one.]
15.) A resting wheel has a constant torque applied to it. After the first 10 seconds, the wheel has turned through an angular displacement of 1.2 radians. After the first 20 seconds, the angular displacement will be:
a.) 2.4 radians. [The relationship we want to use here is $\Delta \theta=\omega_{1} t+.5 \alpha t^{2}$. With the
initial angular velocity equal to zero, the angular displacement is a function of $t^{2}$. This means that doubling the time should quadruple the angular displacement. 1.2 radians quadrupled is 4.8 radians. This response is false.]
b.) 4.8 radians. [This is the one.]
c.) 7.2 radians. [Nope.]
d.) None of the above. [Nope.]
16.) A graph of the position function for a body oscillating with a frequency of $1 /(4 \pi)$ radians per second in simple harmonic motion is shown to the right. The equation that best describes the motion is:
a.) $y=20 \sin (.5 t-\pi / 2)$. [The amplitude of this graph is 10 meters, which eliminates Responses $a$ and d.]
b.) $\mathrm{y}=10 \sin ([1 /(4 \pi)] \mathrm{t}-5)$. [This has the correct amplitude. The angular frequency will be $2 \pi$ times the frequency,
 or $(2 \pi)(1 /(4 \pi)=.5$ cycles/second. This eliminates Response b.]
c.) $\mathbf{y}=10 \sin (.5 t+3 \pi / 2)$. [This has the correct amplitude and angular frequency. The easiest way to determine the phase shift in this case is to eyeball it. If you shift a sine wave by $\pi / 2$ radians to the right, you will have a graph defined by $x=A$ at $t=0$. If you shift a sine wave by $\pi$ radians to the right, you will have a graph defined by $x=0$ going negative at $t=0$. If you shift a sine wave by $3 \pi / 2$ radians to the right, (this is the phase shift for the function we are testing), you will have a graph defined by $x=-A$ at $t=0$. In fact, that is exactly what our graph is doing at $t=0$, so this function must be O.K.]
d.) $y=20 \sin ([1 /(4 \pi)] t-\pi / 2)$. [Nope.]
e.) None of the above. [Nope.]
17.) A sound wave moving at $330 \mathrm{~m} / \mathrm{s}$ has a frequency of 220 Hz . Its wavelength is:
a.) $2 / 3$ meter. [Using the relationship $\mathrm{v}=\lambda v$, the wavelength is found to be 1.5 meters. The incorrect value of this response is the inverse of that value.]
b.) 1.5 meters. [This is the one.]
c.) 66,000 meters. [Not likely, but quite a number (it is the product of 220 and 330 ). This is a false response.]
d.) None of the above. [Nope.]
18.) A mass $m_{1}$ sits on top of a second mass $m_{2}$ which sits on a frictionless surface. The coefficient of static friction between the two is .7. A force $F$ is applied to the top mass ( $m_{1}$ ).
a.) The maximum force $F$ that $m_{1}$ can experience without slipping over $m_{2}$ is $\mu_{\mathrm{s}} \mathrm{m}_{1} g$. [This is tricky. The normal force acting on $m_{1}$ is $m_{1} g$, so the static frictional force will be $\mu_{s} m_{1} g$. This means that if $m_{2}$ had been fixed to the ground (i.e., held stationary), the largest non-slip value $F$ could attain would be $\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$. Unfortunately, that's not the situation-- $m_{2}$ isn't fixed to the ground, it resides on a frictionless surface. Therefore, this statement is false.]
b.) The maximum force $m_{1}$ can experience without slipping over $m_{2}$ is $2 \mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$.
[Summing the forces on $m_{1}$ yields $F-\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}=m_{1} a$, or $F=m_{1} a+\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$. To get $a$, noting that "no slippage" means that $m_{1}$ 's acceleration equals $m_{2}$ 's acceleration, we can use N.S.L. on $m_{2}$. Doing so yields the following: $\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}=m_{2} a$, or $a=\left(\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}\right) / m_{2}$. Substituting $a$ into our first equations yields: $\left.F=m_{1}\left[\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}\right) / m_{2}\right]+\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}=2 \mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$. This statement is true $\ldots$ but are there other correct responses?]
c.) The maximum force $m_{2}$ can experience without slipping relative to $m_{1}$ is $\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$. [The only force acting on $m_{2}$ is friction. The maximum static frictional force between $m_{1}$ and $m_{2}$ is, indeed, $\mu_{\mathrm{s}} \mathrm{m}_{1} \mathrm{~g}$. This response is true.]
d.) Both band c. [Yup.]
e.) None of the above. [Nope.]
19.) A spinning skater's angular speed is $2 \mathrm{rad} / \mathrm{sec}$ when her kinetic energy is 20 joules. If the skater's moment of inertia changes by half during the spin, what is her new angular speed?
a.) . $5 \mathrm{rad} / \mathrm{sec}$. [Assuming there are no external torques being applied to the system, which is the case with a spinning body like this, conservation of angular momentum maintains that $I \omega$ must remain the same during the spin no matter how the moment of inertia might change. With a constant $I \omega$, halving $I$ would necessitate the doubling of $\omega$. In short, the solution should be $4 \mathrm{rad} / \mathrm{sec}$. This response is false.]
b.) $2 \mathrm{rad} / \mathrm{sec}$. [Nope.]
c.) $(8)^{1 / 2} \mathrm{rad} / \mathrm{sec}$. [This is an example of one of the ways that A.P. tests can be tricky. Energy was placed in this problem specifically to nudge you toward using conservation of energy in a situation where it is not appropriate (energy is not conserved here). The kinetic energy information can allow you to determine the moment of inertia for the system, but as Response a points out, you don't need that information to think through the problem. If you went for the deception and used conservation of energy on the problem, you undoubtedly came up with an angular velocity of $(8)^{1 / 2} \mathrm{rad} / \mathrm{sec}$. That response is false.]
d.) $4 \mathrm{rad} / \mathrm{sec}$. [This is the one.]
20.) A mass $m$ is attached to a string of length $L$ that is, itself, attached to the ceiling. The mass is drawn to the side and released so that it swings back and forth. At the bottom of its arc, the mass's velocity is $v_{o}$.
a.) The acceleration of the mass when at the bottom of the arc is zero. [If the acceleration were zero, the mass would continue moving in a straight line with velocity $v_{o}$. As that isn't what happens, this statement is false.]
b.) The acceleration of the body when at the bottom of the arc is a function of the body's mass. [The relationship between the inertial mass and gravitational mass of a body holds here. The mass of the object makes no difference in determining the body's acceleration. This statement is false.]
c.) The force on the mass when at the bottom of the arc is $m g$. [In terms of force variables, the force at the bottom of the arc will be upward and equal to $T-m g$. This is also equal to the product of the mass and acceleration, or $m v_{o}^{2} / L$. This statement is false.]
d.) The net force on the mass when at the bottom of the arc is $\left(m v_{o}{ }^{2} / L\right)(j)$.
[There are no forces in the direction of motion, so the mass will free wheel through the bottom. There must be a force perpendicular to the motion and pointing toward the center of the arc, though, or the body would not move in a curved path. The magnitude of the associated centerseeking acceleration has to be $v_{o}^{2} / L$, where $L$ in this case is both the length of the string and the radius of the arc, and the magnitude of that center-seeking force is $m v_{o}^{2} / L$. This statement is true.]
21.) A flatbed truck coasts forward frictionlessly along on a flat surface. The surface between the truck bed and a large, massive crate sitting on the bed is frictional. At $t=0$, you begin to push the crate toward the rear of the bed. At $t=1$ second, the crate leaves the truck.
a.) Between $t=0$ and $t=1$ second, the momentum of the truck is conserved. [If you take the system to be the truck alone, there will be two external forces acting on it. Specifically, you will be applying force to the bed as you traction off of it to push the crate, and there will be friction acting between the crate and the truck. As these external forces exist, the truck's momentum alone cannot be conserved and this response is false.]
b.) Between $t=0$ and $t=1$ second, the momentum of the truck/crate system is conserved, and the truck will have slowed by the time the crate leaves it. [Friction between the truck and crate is an internal force (Newton's Third Law--the frictional force the crate applies to the truck is equal and opposite the frictional force the truck applies to the crate), and the force you apply to the crate is equal and opposite the force your feet apply to the truck (without friction between your feet and the truck bed, you wouldn't be able to push at all). That means you are effectively a part of the system, whether the problem states this or not. As there are no external forces acting on the truck/crate system (at least not in the horizontal), momentum must be conserved in the horizontal and the first part of this response is true. What about the second part? Relative to the ground, the crate has to be moving more slowly than the truck or the two will continue moving with the same speed and the crate won't traverse over the bed's surface. What is slowing the crate, relative to the ground? You are pushing! But as you push the crate toward the rear of the truck, you must also be pushing the truck forward (Newton's Third Law and the presence of friction). As such, the speed of the truck will increase as you push, and this response is false.]
c.) Between $t=0$ and $t=1$ second, the momentum of the truck/crate system is conserved, and the truck will have sped up by the time the crate leaves it. [It looks like this is the one.]
d.) Between $t=0$ and $t=1$ second, the momentum of the truck/crate system is not conserved as you are applying an external force to the crate. [Nope.]
e.) None of the above. [Nope.]
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Additional NOTE about Problem 5: If you include friction, the energy loss down a flat incline is more than the energy loss down a curved incline, assuming the total distance traveled in both cases is the same. This may seem counterintuitive, so I'm taking a moment to explain why. As you know, the amount of work that friction does on a body moving over a frictional surface depends solely upon the size of the frictional force and the distance over which that force acts. The frictional force, on the other hand, depends upon the coefficient of friction (a constant in this case) and the magnitude of the normal force. For the flat incline, the normal force is constant so the frictional force $f_{\text {flat }}$ is constant. For our curved incline the coefficient of friction is the same as in the flat incline case, but the frictional force at the top of the curved incline is small because the normal force at

flat incline friction and curved incline friction aren't the same till here
the top is small. By the same token, the frictional force at the bottom of the curve is large because the normal force at the bottom is large. If, for half of the motion, $f_{\text {curve }}$ was less than $f_{\text {flat }}$ and, for the other half of the motion, $f_{\text {curve }}$ was more than $f_{f l a t}$, then the net effect would be the same as that observed with the flat incline. The problem is that for a flat incline of height $R$ to have the same length as a quarter circle of radius $R$, the angle of the incline has to be $39.5^{\circ}$. The normal force on the curved incline does not become less than the normal force on the flat incline until the body has reached the $39.5^{\circ}$ point. The lesser amount of work done before that point is not countered by an appropriately greater amount of work below the $39.5^{\circ}$ mark, and the net effect does not match the flat incline. And yes, this is very obscure.

